KING'S COLLEGE SCHOOL

ENTRANCE SCHOLARSHIP EXAMINATION

SPECIMEN PAPER

MATHEMATICS

1 1/2 hours

There are SIX questions.

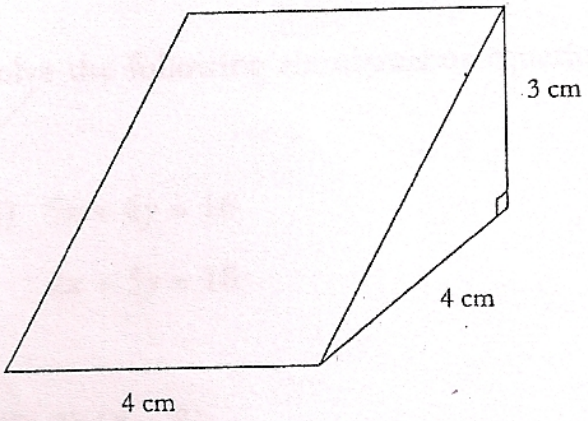
Answer as many as you can, showing sufficient working to make your methods clear, and giving explanations where necessary.

Calculators are allowed.

1. (a) The **solid** in the figure below is a prism, with cross-section a right-angled triangle.

(i) Find the **volume** of the solid.

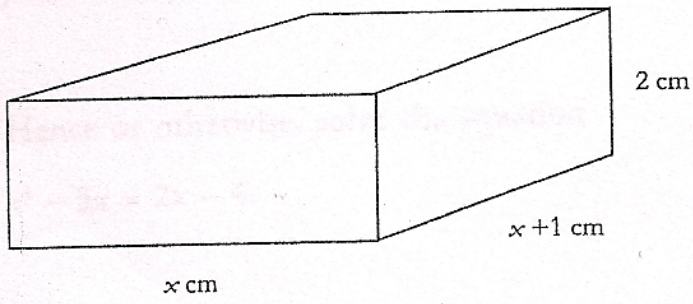
(ii) Find the **total surface area** of the solid.



(b) The **solid** in the figure below is a cuboid.

(i) Find, in terms of x, an expression for the **volume** of the solid.

(ii) Find, in terms of x, an expression for the **total surface area** of the solid.



2. (a) Solve the following equations:

(i) $3x + 12 = 4(x + 2)$

(ii) $4^{(6x - 12)} = 64$

(b) Solve the following simultaneous equations:

(i) $5x + 6y = 16$

$2x + 3y = 10$

(ii) $3^{(x + y)} = 81$

$3x - 2y = 22$

3. (a) Using the graph paper provided, plot the graph of

$y = 2x - 4$ for $-1 \leq x \leq 5$.

(b) **On the same set of axes**, plot the graph of

$y = x^2 - 3x$ for $-1 \leq x \leq 5$.

(c) Hence or otherwise, solve the equation

$x^2 - 3x = 2x - 4$.

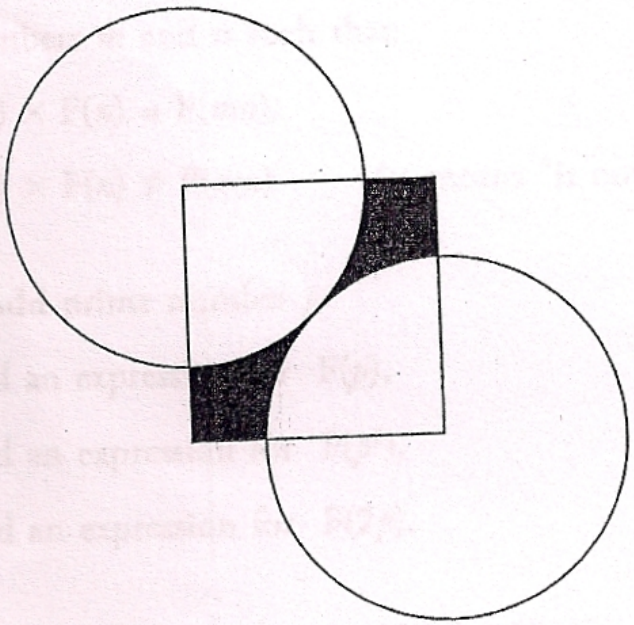
(d) Find a pair of values, x and y, satisfying

$x^2 - 3x < y < 2x - 4$.

4. (a) Given that a circle has circumference 3π m, find its area.

(b) Given that a circle has area $\frac{\pi}{2}$ m^2, find its circumference.

(c) In the figure below, the square has a side of length 2 m.
The two circles are touching.
Each circle has centre at a vertex of the square.



(i) Find the area of the shaded region.

(ii) Find the perimeter of the shaded region.

5. For a positive whole number n, $F(n)$ is defined to be the sum of all the **factors** of n.

For example, $F(10) = 1 + 2 + 5 + 10 = 18$.

(a) Evaluate:

(i) $F(7)$,

(ii) $F(12)$.

(b) Find numbers m and n such that:

(i) $F(m) \times F(n) = F(mn)$,

(ii) $F(m) \times F(n) \neq F(mn)$ ($\neq$ means "is not equal to").

(c) For an **odd** prime number p:

(i) Find an expression for $F(p)$,

(ii) Find an expression for $F(p^2)$,

(iii) Find an expression for $F(2p)$.

(d) Find a single-digit number n satisfying $F(n) = 2n$.

(e) Evaluate the following sequence of values:

$F(1)$, $F(2)$, $F(4)$, $F(8)$.

(f) By considering the sequence of values obtained in part (e):

(i) Find the value of $F(32)$,

(ii) Find the value of $F(128)$,

(iii) Find an expression for $F(2^n)$.

6. **Positive whole numbers** a, b and c are such that $a < b < c$.

(a) Find **all** possible values of a, b and c that satisfy:

$a \times b \times c = 24$

(b) Find **all** possible values of a, b and c that satisfy:

$a \times b + c = 12$

(c) Find **all** possible values of a, b and c that satisfy:

$a + b + c = 12$